Lecture #1

September 3rd, 2019

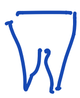
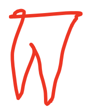
Counting and Combinatorial Analysis.

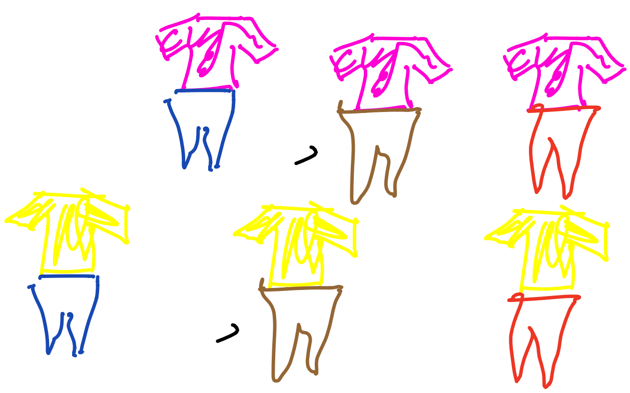
- Probability is the study of quantifying the liklihood of a random event.
- In many such situations, we need to know how many possible events can occur.
  - ↳ Ex: A lottery consists of choosing four digits from 0-9. How many possible tickets are there?
- The general methods (techniques) for counting discrete (not discreet) events is called <u>combinatorial analysis</u>

Basic Counting Principle

Suppose two experiments are being performed, and suppose the first experiment has n possible outcomes and the second experiment has m possible outcomes. Then the number of outcomes for the experiments together is mn.

Basic Example!

Suppose that I have two shirts  and , and three pairs of pants, , , . Say each "experiment" is me choosing a shirt or a pair of pants. How many outfits do I have?



$$2 \times 3 = 6$$

outfits

Generalized Counting Principle.

Suppose there are r many experiments, with the experiment i having n_i outcomes. Then the total number of possible outcomes is

$$n_1 n_2 \cdots n_r := \prod_{i=1}^{r} n_i$$

Ex: How many possible lottery tickets are there if you choose ~~#~~ four digits from 0–9?

$$\underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 10^4 = 10{,}000 \text{ tickets.}$$

10 possibilities for each one (0 ... 9)

Real Life Example:

Lotto max: each ~~...~~ game consists of 7 numbers chosen from 1–50. So the number of possible "games" are $50^7 \approx 781$ billion.

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Permutations (1.3)

Suppose you have some objects. How many ways can you arrange them in a row?

- Each such rearrangement is called a <u>permutation</u> (because you permute them).

<u>Ex:</u> How many numbers can be formed out of 1, 2, 3? We can just write them out:

$$\underbrace{123,\ 132,\ 213,\ 231,\ 312,\ 321}_{6 \text{ of them!}}$$

Now lets consider a more general situation: Say we have a box of n things and we want to arrange them:

$$\underbrace{\boxed{1,\ 2,\ \ldots,\ n}}_{n \text{ things}} \qquad \underbrace{\underset{1}{\_\_}\ \underset{2}{\_\_}\ \underset{3}{\_\_}\ \cdots\ \underset{n-1}{\_\_}\ \underset{n}{\_\_}}_{n \text{ spots.}}$$

For the first spot, we can choose anything; so there are n possible choices.

$$\underset{1}{\underline{\ n\ }}\ \underset{2}{\_\_}\ \underset{3}{\_\_}\ \cdots\ \underset{n.}{\_\_}$$

For the second spot, there are only $n-1$ things left.

$$\underline{n} \quad \underline{n-1} \quad \_\_\_ \; \_\_\_ \cdots \_\_\_$$

Going on this way we get that each spot has one fewer option. By the basic principle of counting there are

$n \cdot (n-1)(n-2) \cdots 3 \cdot 2 \cdot 1 = n!$ many options.

Permutations with repeats:

Let's count the number of ways we can rearrange the letters in

banana

If we label each of the letters

$$b_1\, a_2\, n_3\, a_4\, n_5\, a_6$$

so that they are all distinguishable, then there are $6!$ different permutations. However,

$$b_1 a_2 n_3 a_4 n_5 a_6 \text{ and } b_1 a_2 n_5 a_4 n_3 a_6$$

are the same!

Suppose now we fix the b and the a's and we only move around the n's. Then there are $2! = 2$ possible arrangements.

If we fix the b and the n's and just move around the a's then there are $3!$ different arrangements.

All together, there are $2!\,3!$ such arrangements where we only swap a's with a's and n's with n's.

Since these permutations are the "same" we cancel them out: $\dfrac{6!}{3!\,2!} = 60$.

General Fact: Given n object of which n_i are alike, $\sum_{i=1}^{k} n_i = n$, then the number of permutations is

$$\frac{n!}{n_1!\,n_2!\cdots n_k!}$$